Forum for Agricultural Research in Africa, Accra Ghana
and
Center for International Food and Agricultural Policy
University of Minnesota, USA

Growth, Structural Change and Total Factor Productivity
in
Eight African Countries

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Study of Growth, Structural Change and Total Factor Productivity in Eight African Countries

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Acronyms

ADMARC	Agricultural Development and Marketing Corporation
FAO	Food and Agriculture Organization of the United Nations
FMB	Farmers' Marketing Board
GDP	Gross Domestic Product
GFCF	Gross Fixed Capital Formation
GTAP	Global Trade Analysis Project
IFPRI	International Food Policy Research Institute
TFP	Total Factor Productivity
TOT	Terms-of-Trade
VAD	Value Added
WDI	World Development Indicators

EXECUTIVE SUMMARY

Introduction

The Comprehensive Africa Agriculture Development Program (CAADP) encourages African governments to increase resources going to, and benefitting, the agricultural sector. Countries accepting the CAADP pledge agree to allocate 10 percent of government spending to the agricultural sector, and commit to improving rural infrastructure. One desired outcome of this investment is for agricultural productivity to double by 2030 and to improve farmer access to markets. This study presents a synthesis of economy-wide and sectoral productivity growth in eight African countries: Burkina Faso, Cameroon, Malawi, Morocco, Nigeria, Tunisia, Uganda and Zambia. Each country study was headed by an economist, native to the country, and measured economywide productivity growth (defined by total factor productivity growth – see section 2.1) and productivity growth for agriculture, manufacturing and services. Each country leader took a close look at sector contributions to country level economic growth and used his or her country specific history to contextualize the evolution of productivity and growth over time.

Methodology and Data

Each country defines (total) factor productivity growth as the difference between growth in gross domestic product (GDP) and the weighted sum of contributions from growth in the capital stock, labor force and cultivated area (see section 2.1). Each country study uses growth accounting to estimate economywide total factor productivity (TFP), with the analyses based on an underlying Cobb-Douglas production function satisfying constant returns to scale and Hicks-Harrod neutral technological change.

The primary data source for each study was the World Bank's World Development Indicators (WDI) database. The WDI provided time series data on: GDP, gross fixed capital formation and the labor force – aggregate and sector levels, adjusted savings, population, and sector value-added. 2007 factor shares for labor, capital and land are taken from GTAP except for Morocco which is based on own data. For all but one country, the data series began in 1970 and ended in 2012 or 2013. WDI labor data was scant for several countries, and in those cases sectoral labor statistics were collected at national statistic offices.

Missing data (primarily sector labor force levels) were replaced using simple regressions. Relevant deflators were used to convert all data to 2007 as the benchmark for the study. Factor contribution to growth was estimated using two different methods for the economy wide analysis and the sectoral analysis: with exogenous depreciation and with endogenous depreciation. Later sectoral contribution to growth, sectoral net capital formation, capital stock and sectoral TFP were estimated. The accounting method is carried out in a manner wherein sectoral weighted TFP estimates are consistent with the economy-wide TFP estimates.

Results

Post-2000 growth in aggregate and per capita GDP of each country exceeds the world average. Most countries succeeded in decreasing the share of persons living in poverty over the period. However, in spite of this performance, the head count of persons living in poverty has increased. The implication, here, is there is a potential problem with income distribution in our sample of countries.

As measured in this study, if beginning the growth measurement in 2010, only Nigeria would reach the goal of doubling agricultural productivity growth by 2030. Cameroon's agricultural productivity would have doubled by 2035, while Morocco's would have doubled by 2039. Average agricultural TFP growth in Malawi was negative, and hence is difficult to predict how long it would take to double productivity by increases in capital and labor inputs alone. Finally, Tunisia and Uganda would take upwards of 50 years to double productivity.

This summary report extends the country studies by linking TFP results to poverty rates. Results suggest agricultural TFP is strongly correlated with poverty levels. The initial results also suggest TFP growth levels in agriculture, manufacturing and services are all correlated with the decrease in the share of rural residents living in poverty. The results suggest that agricultural TFP growth is highly correlated with poverty reduction, but agricultural TFG growth is influenced by growth in manufacturing and service sector TFP.

1 INTRODUCTION

The African continent is a vast land mass, comprised of over fifty countries and home to more than 1 billion men, women and children. Most of these countries became independent states after 1960, and in spite of the myriad challenges encountered across the continent, many of its countries have managed to realize economic growth rates that surpass the world average. For example, the average annual rate of growth in world gross domestic product (GDP) was 3.04 percent between 1970 and 2014, while the corresponding growth in GDP for Sub-Saharan Africa was 3.17 percent (World Bank, World Development Indicators - WDI). The average rate of growth in Sub-Saharan Africa's GDP per capita, however, was significantly lower than the corresponding rate of growth in world GDP per capita – 0.038 percent for Sub-Saharan Africa as compared to 1.146 percent for the world (WDI). Africa's meager growth in GDP per capita, of course, reflects the fact that its population growth has outpaced its income growth. This income/population growth tradeoff lay at the core of the African development problem: without increasing income per person, and without viable income transfer schemes, how can an economy pull its poor out of poverty.

Tackling poverty with a relatively fast growing population has presented Africa-centric policy makers and politicians quite a challenge. One of the major problems faced by the poor is access to food. Realizing the need to improve access to food, in 2003 the African Union launched the Comprehensive Africa Agriculture Development Program (CAADP). A major objective of CAADP is to encourage African governments to increase resources going to, and benefitting, the agricultural sector. It is hoped that increasing food production and improving rural infrastructure and agricultural marketing would improve food availability and access in order to eliminate – or at least decrease significantly – malnutrition. Countries accepting the CAADP pledge agree to allocate 10 percent of government spending to the agricultural sector and commit to improving rural infrastructure.

One of the CAADP goals adopted in the Malabo Declaration of July 2014 is to double agricultural productivity by 2025 (AUC, 2014). The African Union also supports the CAADP platform as a mechanism to integrate data into policy design and decisions. This report summarizes the results of eight country studies of agricultural, industrial and service sector productivity: Burkina Faso, Cameroon, Malawi, Morocco, Nigeria, Tunisia, Uganda and Zambia, with each country study headed by an economist native to the region.

In addition, each study develops baseline measures of manufacturing and service sector productivity. We follow this approach because it helps us better understand agriculture's role in economic growth and overall economic productivity.

The studies also examine agriculture's role in structural adjustment (i.e., its relative importance in producing value added in an economy and labor dynamics).

For example, increased factor productivity in industry and services tends to increase average household income through increased employment and returns on investment. If access to food was a problem, increased income levels will tend to provide households with increased access to food. On the other hand, increased agricultural productivity helps agriculture compete with the rest of the economy for resources – and improves land rent. As food production grows, downward pressures on food prices tend to occur, which in turn, can encourage a movement of labor out of agriculture and into the industrial and service sectors, where labor productivity tends to be higher than that in agriculture.

This synthesis report is organized as follows. We begin by defining total factor productivity – the primary productivity measure used in this report – and introducing two other concepts useful when discussing economic growth and development (capital deepening and structural change). The third section gives an overview of the (general) data sources used in the studies and discusses the major data challenges faced by country teams. The fourth section summarizes the empirical productivity results, and the patterns of structural change experienced by the countries. The fifth section concludes and offers suggestions on how to improve on the type of productivity study we pursued. An appendix provides details of the theory used in the country studies.

2 Factor Productivity and Economic Growth

Productivity measurement takes one of two forms: partial factor productivity and total factor productivity. A partial measure of productivity is the ratio of output produced (e.g. tons of maize) to the level of an input used (e.g., hectares of land or labor in person-days). Partial productivity measures have the advantage of being easy to understand, and provides insights into the efficiency of a single input in a production process. These measures, however, can fall short in conveying the full story of productivity changes, as the lone input is often not solely responsible for a change in output. An increase in labor productivity for example, may be due to the use of fertilizer and/or some other input involved in the production process. Total factor productivity (TFP) measures productivity as the ratio of an index of agricultural output to an index of agricultural inputs. It gives a measure of the efficiency of all inputs involved in a production process. In this regard, TFP is regarded as a more complete measure of productivity than partial measures.[1]

When discussing TFP, one needs to be clear on whether they are talking about changes in productivity levels, or rates of change in productivity. To illustrate the difference between changes in productivity levels and rates of change in productivity, consider the following production function

[1]Diewert (undated) defines total factor productivity of a firm (or industry or group of industries) as the real output produced by the firm over a period of time divided by the real inputs used by the firm over the same period of time.

(1) $$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}$$

Here, Y_t represents output produced in year t, while K_t and L_t are the respective amounts of capital and labor used to produce that output. The parameter α is a measure of how much output changes given a small change in capital use (all else constant), as well as a measure of the share of capital's cost in producing a unit value of output. Similarly, $1-\alpha$ is a measure of how much output changes (all else constant) given a change in labor use, as well as a measure of the share of labor's cost in producing a unit value of output.

The parameter A_t is a "level" measure of technology: it is an index of output not explained by capital and labor. To see this, note that $K_t^{\alpha} L_t^{1-\alpha}$ is the amount of output explained by capital and labor. The amount of output not explained by the two inputs is equal to $A_t = \frac{Y_t}{K_t^{\alpha} L_t^{1-\alpha}}$. If the amount of capital and labor used remained the same, but output increased, then necessarily A_t would have increased. In such a case, we would say the *level of the technology* (*or technical efficiency*) increased.

2.1 Total factor productivity and economic growth[2]

To represent technology growth, we typically begin by first taking the log of equation (1) to get

$$\ln Y_t = \ln A_t + \alpha \ln K_t + (1-\alpha) \ln L_t$$

In other words, we can divide output into three separate components: technology's contribution to output, $\ln A_t$, capital's contribution to output, $\alpha \ln K_t$, and labor's contribution, $(1-\alpha)\ln L_t$. We then subtract $\ln Y_{t-1}$ from $\ln Y_t$ to get

$$\ln Y_t - \ln Y_{t-1} = \ln A_t - \ln A_{t-1} + \alpha(\ln K_t - \ln K_{t-1}) + (1-\alpha)(\ln L_t - \ln L_{t-1})$$

and recall that the difference in logarithms, $\ln Y_t - \ln Y_{t-1}$, is approximately equal to the rate of growth in output, Y_t, i.e.,

$$\ln Y_t - \ln Y_{t-1} \cong \frac{Y_t - Y_{t-1}}{Y_{t-1}}$$

Finally, the reader can verify that using the above logarithm rule and rearranging terms yields the following growth relation

[2] The methodology discussed in this section was applied at both the national and sector levels in each country.

(2) $$\ln Y_t - \ln Y_{t-1} \cong \frac{Y_t - Y_{t-1}}{Y_{t-1}} = \alpha \frac{K_t - K_{t-1}}{K_{t-1}} + (1-\alpha)\frac{L_t - L_{t-1}}{L_{t-1}} + \frac{A_t - A_{t-1}}{A_{t-1}}$$

i.e., the rate of growth in output is a linear combination of the (cost share weighted) rate of growth in capital and labor plus the rate of growth in the level of the technology. For example, capital's contribution to growth in output is the term $\alpha \frac{K_t - K_{t-1}}{K_{t-1}}$.

Fundamentally, TFP is typically thought to result from technological innovations and therefore gives a measure of changes in the efficiency of converting capital and labor – represented here by K_t and L_t respectively – into output, Y_t. The long-term growth of an economy is to a large extent attributable to the growth of TFP or technological progress (Kim and Lau, 1994). Typically, one does not directly observe TFP, e.g., A_t: instead, it is measured by creating an index based on observable output(s) and inputs. One of the most famous technology measures is embedded in the growth expression (2), where a simple rearrangement of terms yields

$$\frac{A_t - A_{t-1}}{A_{t-1}} = \frac{Y_t - Y_{t-1}}{Y_{t-1}} - \alpha \frac{K_t - K_{t-1}}{K_{t-1}} - (1-\alpha)\frac{L_t - L_{t-1}}{L_{t-1}}$$

Here, $\frac{A_t - A_{t-1}}{A_{t-1}}$ is an index known as the *Solow residual*, and is a measure of the growth in output not accounted for by the changes in input use.

Decomposing growth into input and technical components – as in equation (2) – is referred to as growth accounting, and is attributed to Robert Solow (1957). This approach involves putting together detailed accounts of inputs and outputs, aggregating them into input and output indexes and using the indices to calculate a TFP index (Diewert, 1980; Zepeda, 2001). The growth accounting model developed by Solow (1957) has been widely used by development and agricultural economists to measure productivity growth across countries, or to compare the productive efficiency of alternative farming systems (Ehui and Jabbar, 2002).

2.2 Structural change

The major objective of this section is to give an overview of what economists mean by productivity and how productivity is related to economic growth. Readers might also benefit from a short description of two other terms – capital deepening and structural change. An economy experiences capital deepening when the rate of growth in its capital stock is greater than the rate of growth in its labor force. When this happens, the capital to labor ratio increases as an economy evolves. With more capital per unit of labor, labor becomes more productive, and over time we expect to see wages increase (see Roe et al, 2010 Chapter 2).

To understand what we mean by structural change, divide an economy into two activities – agricultural production and non-agricultural production. In the early stages of development, forty percent of a country's GDP for example, might have come from agriculture and the rest from non-agricultural production. Also, sixty percent of the work force might have been engaged in agricultural production. Today, five percent of that country's GDP might come from agricultural production, and two percent of its workforce engaged in agricultural production. This process of workers exiting agriculture to join the non-agricultural sectors, and the share of GDP shifting from agriculture to non-agriculture is referred to as structural change.

Figures 1 and 2 reveal how structural change occurred in Cameroon over the past 45 years. As the economy grows, the share of agricultural labor has declined, albeit the share of income generated by the non-agricultural sector has remained relatively flat. On the other hand, the share of labor in non-agricultural production increased over time, with its share of income relatively flat. This suggests, in Cameroon, agricultural income per unit of labor increased over time, while its non-agricultural income per unit of labor fell. Indeed, figure 3 reveals this pattern of income per unit of labor for agriculture and non-agriculture (manufacturing and services).

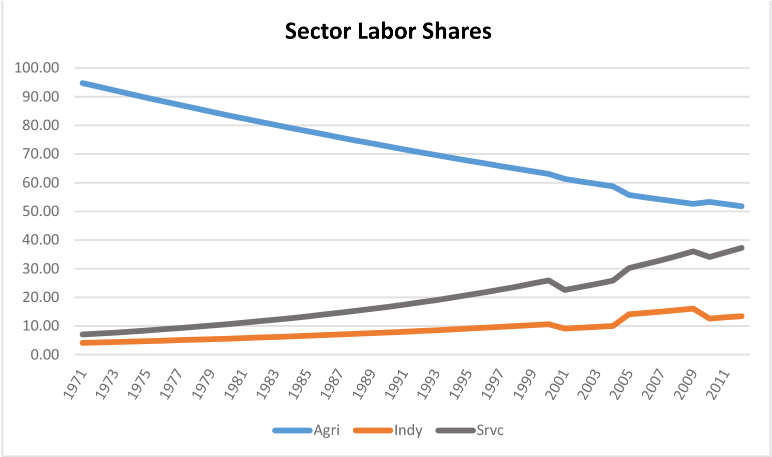

Figure 1. Sector labor shares[3]

[3]Values for 1995 – 2001 estimates provided by Cameroon country study, Tambi et al, 2016.

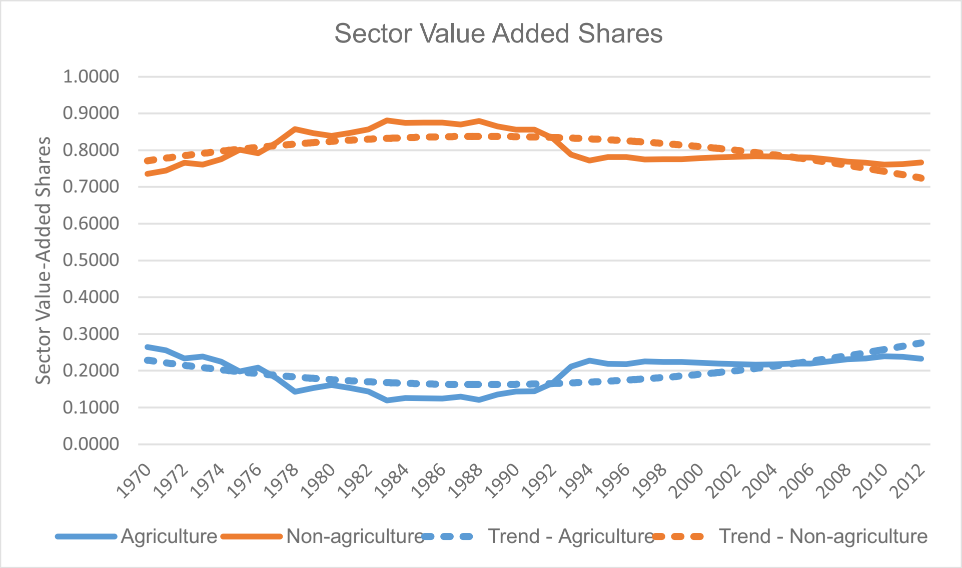

Figure 2. Evolution of value-added shares in Cameroon

This pattern of growth, while encouraging for agriculture and the CAADP goals, does not fit the pattern of growth experienced by most developed countries[4]. This pattern is possibly revealed in the interplay between Cameroon's sector value-added growth and its sector TFP growth. Agricultural value-added grew at an annual average rate of 3.8% over the period 1970 through 2012, while manufacturing and service sector value-added grew at an annual average rate of at least 4.3% over the same period. Cameroon agricultural TFP growth, however, averaged 2.7% a year over the period, while the average annual rate of growth in its manufacturing and service sector TFP was -1.0% and -0.6% respectively: capital and labor in agriculture became increasingly more productive over time (the marginal increase in output was greater than the marginal increase in input use), while capital and labor in manufacturing and services became less productive over time. These relative TFP growth rates suggest that over time, agriculture has been quite successful competing for resources (e.g., capital and new technologies). On the other hand, while manufacturing and services have been pulling labor out of agriculture, technical change has been slower – perhaps slower than the rate at which labor is moving from agriculture into the non-agricultural sectors.

[4]Herrendorf et al. (2013) argues most developed countries experience an exit of labor from agriculture into non-agriculture (observed in Cameroon), and an increase in labor productivity in both agricultural and non-agricultural production (not observed in Cameroon).

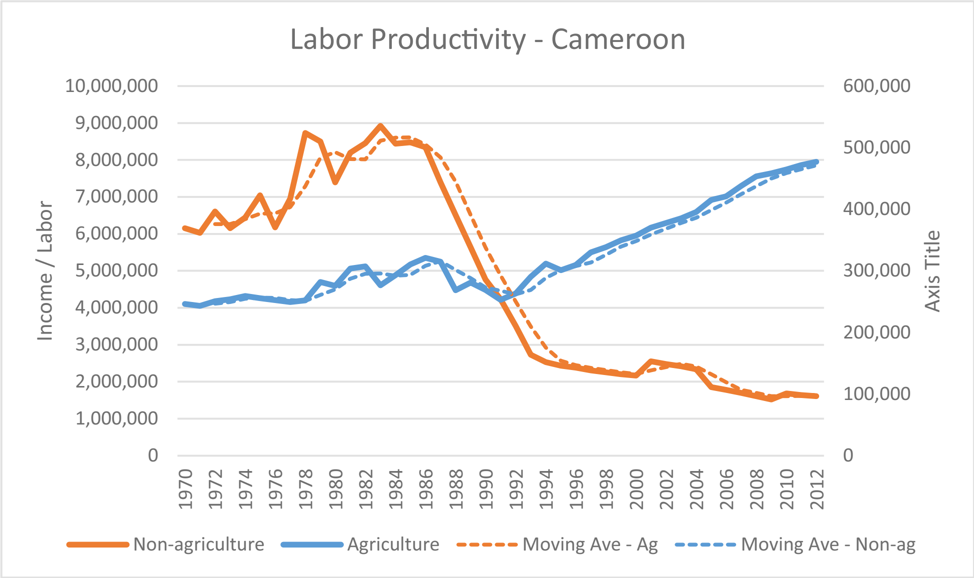

Figure 3. Evolution of labor shares in Cameroon

The lesson to draw from structural change dynamics is that a focus on agricultural TFP, while a useful exercise, is better contextualized when placed in an economy-wide setting. This has important implications for understanding the evolution of income levels and growth rates in the sense that resource movement is a significant source of growth through sector reallocation (Gollin, 2010).

3 Data

Each country study drew its' data from various secondary sources, the main source being the World Bank's World Development Indicators (WDI). For most countries, the WDI provided the following time series: GDP; population and total labor force; agricultural, industrial and service sector value added; official exchange rates;[5] and employment in agriculture, industry and services. Factor shares were mostly obtained from GTAP and IFPRI publications. Sector employment data tended to be a problematic data series, and in several countries were missing for several years. For employment and other missing data, country teams used either linear regression, or employed three-year to five-year moving averages to estimate/predict missing variables. The WDI provided investment data, in this case, gross fixed capital formation (GFK), which was used to create an economywide and corresponding sectoral capital stock series for each country.

Data on depreciation rates are scant or non-existent for most developing countries. Data on the service life of capital assets are also not readily available (Nehru and Dhareshwar, 1993). For these reasons, following Nehru and Dhareshwar (1993), most countries assumed a single exogenous depreciation rate of either 3.5 percent or 4 percent. Depreciation rates were also estimated using WDI data on adjusted savings (in current U.S. dollars) in order to assess the sensitivity of the analysis to the depreciation assumption. See appendix, section 10.2.2 for details.

[5]Exchange rates are implicit in the base year (2007) but, like all other prices, held constant thereafter.

The structure of each country study followed closely the format of Roe, Smith and Choi (2015). Finally, TFP measures estimated using growth accounting can be sensitive to initial capital stock levels: if the initial capital stock level is “small” while gross fixed capital levels are large, then the capital stock growth rates will be large, putting downward pressure on TFP growth estimates. For this reason, several methods were used to estimate initial capital stock levels. For each country, capital stock data are, is in most cases, missing and have to be estimated. The method of estimation has direct influence on the results as all the other computations that involve capital stocks including capital stock growth rates, the rate of growth of capital, the capital-labor ratio, the Solow residual and the capital-output growth rate would be affected. The initial capital stock was obtained using the “Kehoe Method” explained in Equations (15) to (23) in Appendix 2.

4 Summary of Country Reports

4.1 GDP and TFP growth

Each of the countries in this study gained its independence sometime between 1956 (Morocco and Tunisia) and 1964 (Malawi and Zambia). Almost all reports divided its economic transition into three main periods: post-independence, structural adjustment and post structural adjustment. Here, we roughly assign post-independence to the years 1960 – 1980, structural adjustment to the years 1980 – 1999, and post structural adjustment to the remaining years. In terms of economic growth, with the exception of Nigeria, these countries performed reasonably well after independence, while the structural adjustment periods yielded less than ideal economic performance. With the exception of Tunisia, the countries seemed to fair reasonably well after year 2000.

Table 1 presents average annual real GDP growth over 1970–2013 and 2000–2013. In both periods, each country outperformed the world in terms of average GDP growth. With the exception of Burkina Faso, Morocco and Tunisia, average GDP growth per capita between 1970 and 2013 was lower than the world average. Post-1999, all but Cameroon and Malawi outperformed the world average in GDP growth per capita.

Table 1. Average annual real GDP growth

Country	GDP and Population Growth Rates					
	1970 - 2013			2000 - 2013		
	Aggregate GDP	Per capita GDP	Population	Aggregate GDP	Per capita GDP	Population
World Average	0.0316	0.0156	0.0157	0.0291	0.0165	0.0125
Burkina Faso	0.0477	0.0212	0.0261	0.0632	0.0297	0.0294
Cameroon	0.0382	0.0098	0.0281	0.0354	0.0090	0.0261
Malawi	0.0416	0.0110	0.0304	0.0415	0.0125	0.0287
Morocco	0.0451	0.0270	0.0172	0.0461	0.0340	0.0111
Nigeria	0.0392	0.0123	0.0266	0.0798	0.0516	0.0268
Tunisia	0.0486	0.0305	0.0177	0.0397	0.0293	0.0101
Uganda	0.0446	0.0094	0.0326	0.0675	0.0323	0.0341
Zambia	0.0319	0.0026	0.0294	0.0702	0.0407	0.0283

Source: World Bank, WDI

Table 2 summarizes the average annual sector GDP and TFP growth rates of each country since 2000. All but Cameroon manufacturing and Zambian agriculture averaged sector value-added growth rates equal to or greater than 2.3%. Zambia is the only country in which agricultural growth fairs poorly with a negative growth rate (-0.6%). In Cameroon and Malawi, agricultural GDP growth averaged 3.9% and 3.1% respectively, while the Malawi manufacturing and service sectors average almost 5% growth over the period. Nigeria, Uganda and Zambia recorded the most impressive overall GDP growth rates, and in each of these countries, manufacturing and services appear to be fairing quite well.

Table 2. Average annual sector GDP and TFP growthrates (2000 – 2013)

Country	GDP				TFP			
	Total	Ag	Manf	Serv	Total[6]	Ag	Manf	Serv
Burkina Faso	0.060	0.048	0.060	0.069	0.010	0.017	-0.004	-0.007
Cameroon	0.035	0.039	0.007	0.055	0.007	0.023	-0.020	-0.006
Malawi	0.042	0.031	0.049	0.048	-0.049	-0.029	-0.093	-0.062
Morocco	0.046	0.056	0.026	0.048	0.010	0.019	0.012	0.012
Nigeria	0.080	0.091	0.049	0.104	0.021	0.038	-0.029	0.063
Tunisia	0.040	0.023	0.027	0.052	-0.003	0.014	-0.068	0.005
Uganda	0.067	0.023	0.081	0.076	0.022	0.014	0.022	0.023
Zambia	0.070	-0.006	0.093	0.070	0.029	-0.016	0.014	0.047

Source: Results from country reports; ; Ag = agriculture, Manf = manufacturing, Serv = services

All but Malawi and Tunisia experienced positive economy-wide TFP growth. TFP growth for agriculture and services were quite respectable for most countries, while TFP growth in manufacturing was negative in four of the countries. Malawi exhibits a negative economy-wide and sector TFP growth rate, while in Cameroon, growth in manufacturing and services is negative. Positive GDP growth combined with negative TFP growth

[6]The sector-share-weighted sum of sector TFP growth rates should equal economywide TFP growth. This identity holds at each point in time, but averaging sector TFP values over 2000-2013 obfuscates this identity.

suggests sector growth was primarily due to increased input use – with the weighted percentage change in input use being larger than the corresponding percentage change in value-added. A major implication of this phenomenon is the average productivity (e.g., wage per unit of labor) likely falls over time.

Bosworth and Collins (2008) argue that agricultural TFP growth was a major source of economic growth in India and China over a period of 25 years. Inspection of Table 2 shows that agricultural TFP is higher than economy-wide TFP in five of the eight countries. These results are consistent with findings by Martin and Mitra (2001), who estimate TFP growth for agriculture in 49 countries and for manufacturing in 38 countries, and found average agricultural TFP growth was higher in over half the countries.

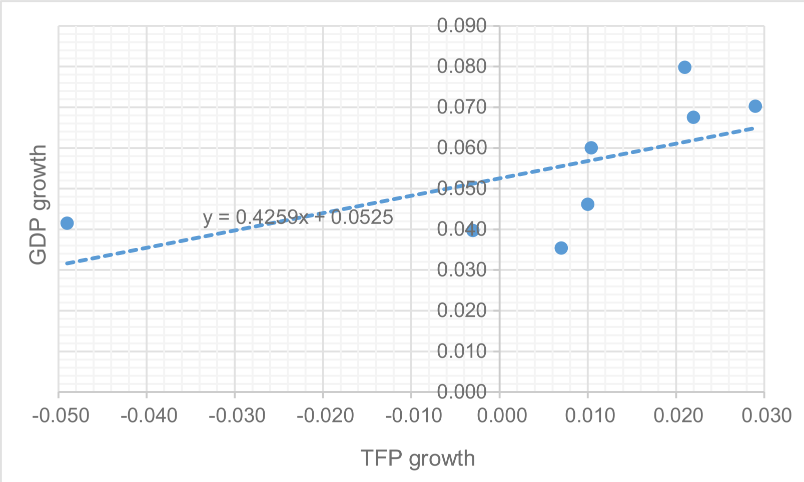

Figure 4. Scatter plot of aggregate GDP and TFP growth

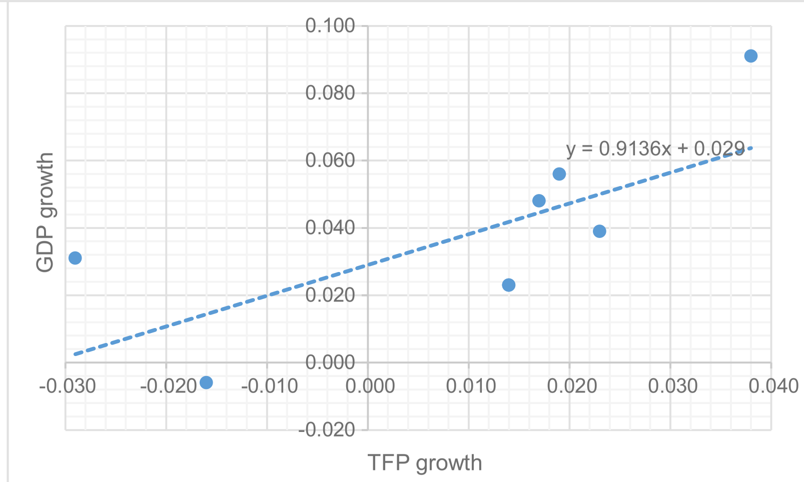

Figure 5. Scatter plot of agricultural GDP and TFP growth

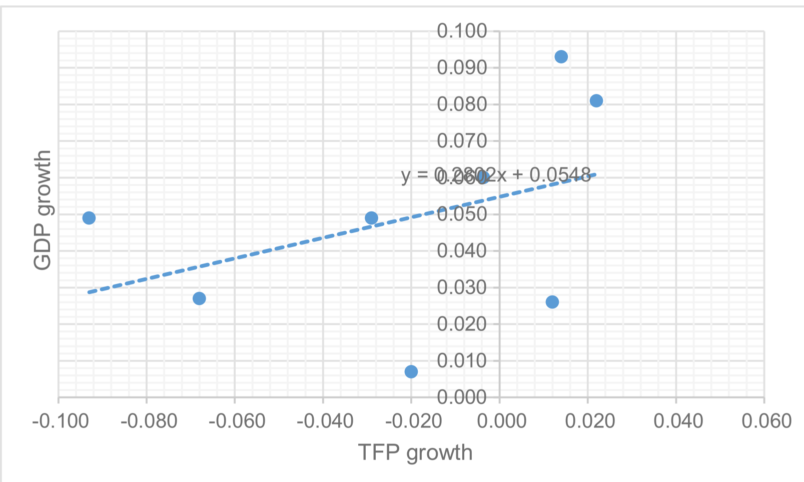

Figure 6. Scatter plot of manufacturing GDP and TFP growth

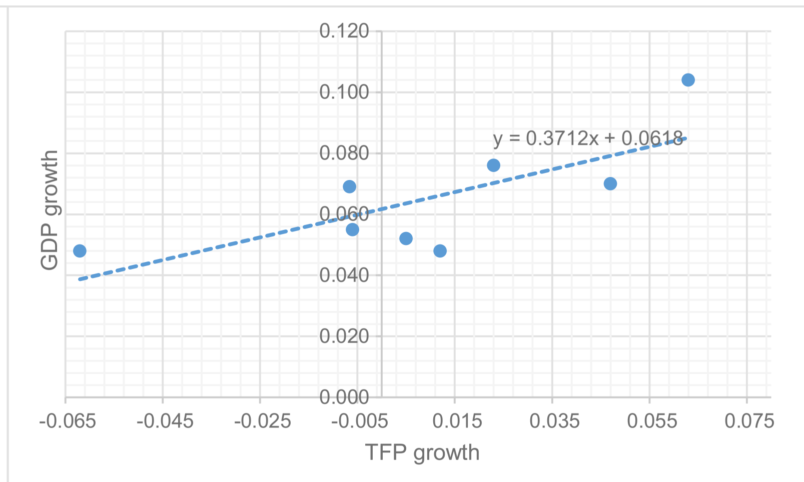

Figure 7. Scatter plot of service GDP and TFP growth

Figures 4 through 7 use the data in table 2 to graphically show the associations between GDP and TFP. The figures reveal that between 2000 and 2013 average annual TFP growth is positively associated with average annual GDP growth: the plots suggest this pattern holds for aggregate level data, as well as with corresponding growth rates for agriculture, industry and services. This relationship exists even though some countries experienced negative TFP growth (while enjoying positive economic growth). Malawi and Zambia were the only countries to experience negative average annual growth in economy-wide TFP and still experienced positive growth in economy-wide value added. As observed above, these results suggest that in each of these two countries, the (weighted) average rate of growth in its stock of capital and labor was larger than the average rate of growth in its GDP.

In Morocco, Malawi, Tunisia and Cameroon, agricultural TFP growth exceeds that of both manufacturing and services. Countries ranking lowest in total GDP growth, Cameroon, Tunisia and Malawi, experienced the lowest rate of growth in service sector GDP. Together, these results leave the manufacturing sector as tending to be the poorest performer for obtaining efficiency gains in these economies – with agriculture and service tending to "pull up" economy-wide growth in value added.

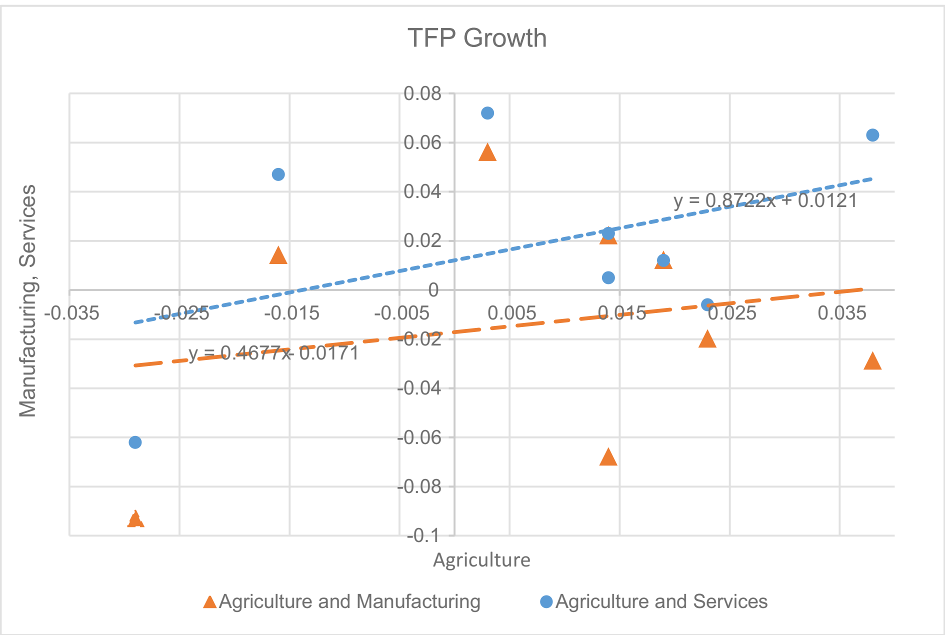

Figure 8. Scatter plots of growth in agricultural and manufacturing TFP, and agricultural and service TFP

Figure 8 suggests agricultural TFP growth is directly related to TFP growth in manufacturing and services. The dotted blue line is the linear model associated with regressing manufacturing TFP growth on agricultural TFP growth, while the dashed orange line is the linear model associated with regressing service TFP growth on agricultural TFP growth. In neither model is the slope coefficient statistically significant. Still, a visual inspection of the plots shows six of the eight observations are consistent with a positive, direct relationship between agricultural and service sector TFP growth (observations in the first and third quadrants of figure 8). At this point, we simply note the data suggests technological spillovers are occurring across the sectors. An example of such spillovers is when improvements in the delivery of administrative, transportation or electrical services also increase the productivity of labor and capital in agriculture.

As noted above, one of the CAADP goals is for agricultural productivity to double by 2030. The "rule of 70" tells us that if a variable was averaging 2% growth per year, it would take 35 years for the level of that variable to double: 35 = 70/2. Hence, assuming Nigeria's per capita GDP would average 5.0% growth each year for the foreseeable future, it would take 70 / 5 = 14 years for its per capita income to double. Table 3 gives the predicted year in which agricultural TFP will have doubled for each country, assuming we started counting in year 2010.

Table 3. The number of years needed to double agricultural TFP

Country	Agricultural TFP Growth	Years to Double	Doubling Year
Burkina Faso	0.017	41.1	2051
Cameroon	0.023	30.4	2040
Malawi	-0.029	-	-
Morocco	0.019	36.8	2047
Nigeria	0.038	18.4	2028
Tunisia	0.014	50.0	2060
Uganda	0.014	50.0	2060
Zambia	-0.016	-	-

Assuming current rates of growth in agricultural TFP prevailed in the future, only Nigeria would be close to meeting CAADP's goal of doubling agricultural TFP by 2025. If TFP growth rates for Malawi and Zambia are to be believed, additional interventions – e.g., additional infrastructure development, developing irrigation systems – will be needed before either country realizes any significant increase in agricultural productivity. Shortly we will see there appears to be some correlation between TFP growth and poverty reduction, as all countries except Malawi and Zambia realized a decrease in the share of individuals living in poverty over the 25-year period beginning in 1994.

4.2 Structural change

Herrendorf et al. (2013) offer that in a desirable transition growth process, agriculture's share in GDP falls over time and eventually levels out, manufacturing's share increases and often declines a bit before leveling out, and the service sector's share of GDP increases and eventually levels out. Accompanying this process is the movement of labor out of agriculture into manufacturing and services, as capital deepening occurs – the process wherein as the economy grows, the capital stock grows faster than the labor force.

Capital deepening should increase labor productivity over time because, on average, a unit of labor will have more capital to work with as the economy grows. This tends to lead to increases in real wages.

Table 4. Structural change (2000 – 2013)

	Capital Deepening				Sector Share			Labor Share		
Country	Total	Ag	Manf	Serv	Ag	Manf	Serv	Ag	Manf	Serv
Burkina Faso	Y	Y	Y	N	D	Flat	I	D	Flat	I
Cameroon	N	Flat	N	N	I	D	I	D	I	I
Malawi	Y	Y	Y	Y	D	Flat	I	D	Flat	I
Morocco	Y	Y	Y	Y	SD	SD	SI	D	Flat	I
Nigeria	Y	Y	Y	Y	I	D	I	SI	SI	SD
Tunisia	Y	Y	N	Y	D	D	I	D	D/Flat	I
Uganda	Y	Y	Y	Y	D	I	I	I/D	D/I	D/I
Zambia	Y	N	Y	Y	D	I	I	SD	SD	SI

Source: Results from country reports; Y = yes, N = no, D = decrease, SD = slight decrease, I = increase, SI = slight increase, Flat = not much change

Table 4, summarizes the structural change process in our countries. In each country, the service sectors share contribution to GDP increases over the period. This is the case for almost all countries since the mid-90s. Manufacturing's share in GDP increased only in Uganda and Zambia, while agriculture's share fell in each country except Cameroon and Nigeria. Sector labor shares followed the Herrendorf et al. pattern in all countries except Nigeria, and capital deepening occurred in all countries except Cameroon. Hence, except for Cameroon and Nigeria, the structural change patterns in our countries appeared similar to those of a typical middle income country.

Table 4 reveals that accompanying the high rates of growth in manufacturing and services, the share of labor in agriculture fell in all countries except Nigeria and Uganda. It is unclear why Nigerian agricultural labor shares increased, but in Uganda cultivated area increased almost 17% over the thirteen-year period. That urban areas benefitted from economic growth is further indicated given that the share of GDP accruing to services increased in all countries.

4.3 Poverty and TFP

Across the world, a major metric of concern is the poverty rate. One desired outcome of economic growth is a decrease in the number and share of people living in poverty. Table 5 shows that, in addition to performing well in terms of aggregate and per capital GDP growth, between 1994 and 2010 all but Malawi and the Zambia, were successful in decreasing the share of people living on \$1.90 or \$3.10 per day in 2011 PPP (purchasing power parity) dollars.

Table 5. Poverty rates – averages over each period

	Share of population living on $1.90 per day			Share of population living on $3.10 per day		
Country Name	1994 - 1997	2000 - 2005	2006 - 2010	1994 - 1997	2000 - 2005	2006 - 2010
Burkina Faso	0.8306	0.5726	0.5529	0.9239	0.7930	0.8047
Cameroon	0.4808	0.2312	0.2927	0.7227	0.5085	0.5427
Malawi	0.6363	0.7363	0.7091	0.8415	0.9007	0.8764
Morocco	0.0740	0.0618	0.0312	0.2643	0.2556	0.1553
Nigeria	0.6350	0.5346	0.5347	0.8104	0.7851	0.7646
Tunisia	0.1086	0.0532	0.0199	0.2869	0.2015	0.0840
Uganda	0.5960	0.6221	0.4146	0.8337	0.8248	0.6937
Zambia	0.4176	0.5669	0.6443	0.6330	0.7402	0.7887

Source, World Bank, World Development Indicators

Although the share of people living in poverty fell, table 6 reveals economic growth was not very successful in decreasing the total number of people living below the poverty line – the exceptions being Morocco and Tunisia. Hence, although each country has experienced admirable economic growth over the past 15 years, most have been unsuccessful decreasing the absolute number of individuals living in poverty – an outcome almost certainly influenced by the high rates of population growth (see table 1) in our eight countries.

Table 6. Number of individuals living in poverty (in millions)

	On less than $1.90 per day			On less than $3.10 per day		
Country Name	1994 - 1997	2000 - 2005	2006 - 2013	1994 - 1997	2000 - 2005	2006 - 2013
Burkina Faso	8.50	7.15	8.14	9.46	9.91	11.85
Cameroon	6.79	3.93	5.73	10.21	8.65	10.63
Malawi	6.33	8.80	9.87	8.38	10.77	12.20
Morocco	2.02	1.83	0.98	7.23	7.58	4.87
Nigeria	69.74	70.07	80.87	89.01	102.90	115.64
Tunisia	0.98	0.52	0.21	2.59	1.97	0.87
Uganda	12.36	16.07	12.87	17.28	21.31	21.54
Zambia	3.92	6.40	8.46	5.94	8.36	10.36

Source, World Bank, World Development Indicators

Table 7 disaggregates the poverty rates into rural and urban rates, and offers a hint at why the CAADP was initiated: in each country, rural poverty rates ranged from 1.5 to over three times that of urban poverty rates. Given the large number of residents living in rural areas, these ratios translate into quite large numbers of rural poor – over the past nine years, there were between 2 to 17 times more poor who lived in rural areas than in urban areas.

Table 7. Share of population living below the national poverty line

Country	2000 - 2005			2006 - 2013			% change in poverty rate	
	Rural	Urban	RUSPR[#]	Rural	Urban	RUSPR	Rural	Urban
Burkina Faso	0.658	0.222	2.964	0.528	0.252	2.095	-0.198	0.135
Cameroon	0.521	0.179	2.911	0.55	0.122	4.508	0.056	-0.318
Malawi	0.559	0.254	2.201	0.566	0.173	3.272	0.013	-0.319
Morocco	0.251	0.076	3.303	0.144	0.048	3.000	-0.426	-0.368
Nigeria	0.566	0.379	1.493	0.528	0.341	1.548	-0.067	-0.100
Tunisia	-	-	-	-	-	-	-	-
Uganda	0.342	0.137	2.496	0.224	0.096	2.333	-0.345	-0.299
Zambia	-	-	-	0.779	0.275	2.833	-	-

Source: World Bank, WDI; [#]RUSPR is an acronym for the ratio of the (share of) rural to (share of) urban poverty rate.

Post-2000, all but Burkina Faso saw a fall in the share of urban residents living in poverty. All but Malawi and Cameroon saw a decrease in rural poverty rates – data limitations preclude us from commenting on Zambia. At this point, we can only note that Malawi and Zambia averaged negative rates of growth in agricultural TFP since 2000, while Cameroon and Malawi experienced negative rates of growth in manufacturing and service sector TFP.

Figures 9 through 11 show the association between TFP growth and rural poverty reduction (fall in share of individuals living in poverty. If TFP growth helps reduce the share of rural people in poverty, we should see observations in the second and fourth quadrant. In figures 9 and 11, five of the six countries fall in the second or fourth quadrant: Cameroon is the outlier in Figure 9, while Burkina Faso is the outlier in Figure 11 –both countries having negative non-agricultural TFP growth rates. Figure 12 suggests agricultural TFP has a negative relationship with urban poverty reduction, with four of the six countries in the first or fourth quadrants. Not plotted are manufacturing and services relation with urban poverty reduction, which have only 2 and 3 countries, respectively, laying in the fourth quadrant. In any event, figures 8 - 12 suggests agricultural TFP growth has an impact on (or at least has a negative relationship with) poverty reduction, but productivity growth in manufacturing and services are also important forces in poverty reduction.

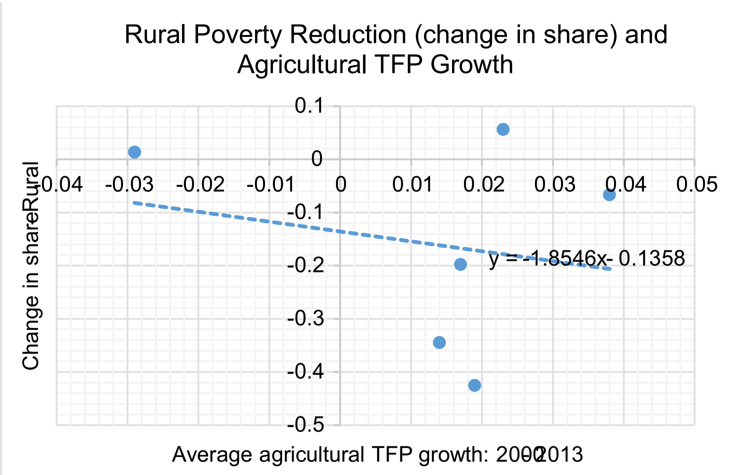

Figure 9.

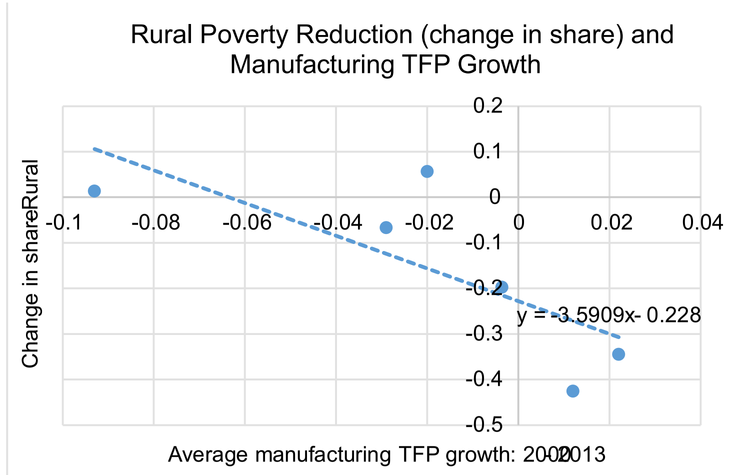

Figure 10.

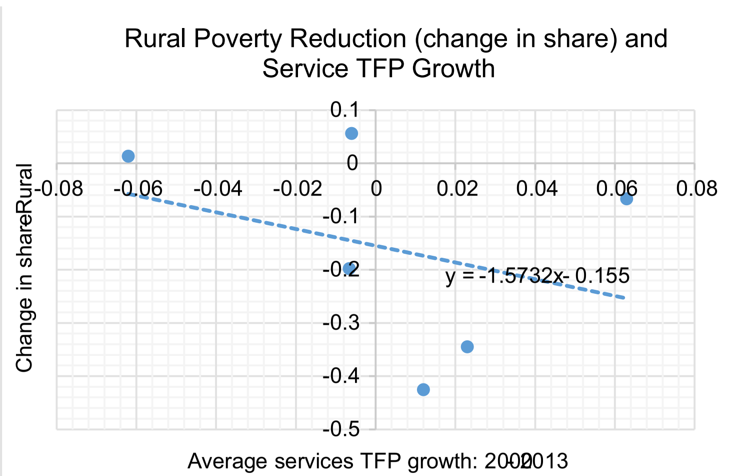

Figure 11.

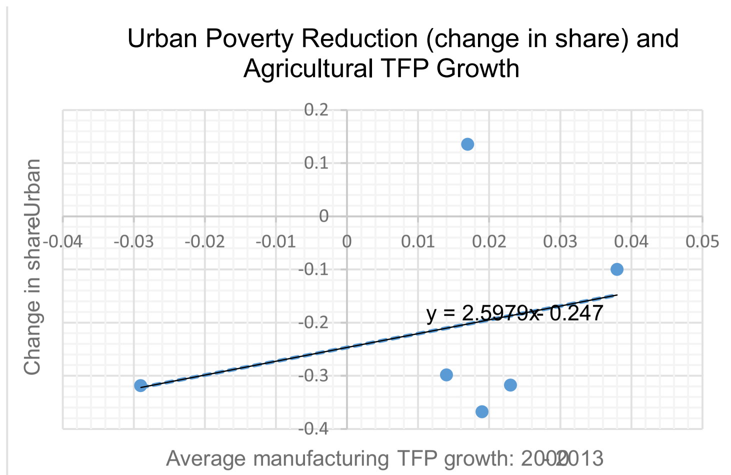

Figure 12.

5 CONCLUSION

The country studies implemented under this FARA productivity project measured total factor productivity growth for three sector aggregates – agriculture, manufacturing and services. The studies are unique in that they introduce a systematic method of constructing sector level capital stock series by combining World Development Indicator data with social accounting matrices and input-output data. These studies also provide what one might call a template for conducting growth diagnostics: measure GDP and TFP growth, search for evidence of capital deepening, ascertain if labor is moving out of agriculture, and uncover other evidence of structural change – e.g., changes in sector contributions to aggregate GDP.

Each country study used their sector level capital stock series to calculate sector TFP growth over time. Aggregate and per capital GDP growth for each country outpaced the corresponding world averages. TFP growth was positive for all countries expect Malawi, and agricultural TFP growth was impressive for all but Malawi and Zambia. Agricultural TFP growth seems to positively correlated with TFP growth in manufacturing and services, although the current summary does not examine whether agricultural TFP growth is "caused" by TFP growth in the other two sectors.

All but Cameroon experienced capital deepening, and all but Nigeria saw the share of labor in agriculture fall over the past 15 years. Structural change occurred in each country, with the service sector's share of GDP increasing over time in each country. In Cameroon and Nigeria, agriculture's share of GDP increased at the expense of manufacturing. In other words, the growth pattern for most of the study countries were similar to that observed by Herrendorf in his study of structural change in OECD countries. That poverty levels increased over time in most countries suggests growth alone may not provide the solution to poverty alleviation that some economists and policymakers had hoped.

Combined with poverty data, the results suggest TFP growth may have direct relationships with rural poverty reduction: with positive (negative) TFP growth in agriculture, manufacturing and services associated with decreases (increases) in the share of rural denizens living in poverty. These preliminary results also suggest only agricultural TFP growth has a direct relationship with urban poverty reduction.

Country results suggest only Nigeria would reach the CAADP goal of doubling agricultural productivity growth by 2030. Cameroon's agricultural productivity would have doubled by 2035, while Morocco's would double by 2039. Average agricultural TFP growth in Malawi was negative, and hence is difficult to predict how long it would take to double productivity. Finally, Tunisia and Uganda would take upwards of 50 years to double productivity. The results also reveal post-2000 growth in the aggregate and per capita GDP of each country exceeds the world average, and in this regard augers well for economic growth across the countries. Again, the poverty data gives rise to concern.

These eight studies constitute an introduction to productivity and structural change in Africa. In our opinion, affordable investments in consulting activities can be replicated to provide insights into the growth dynamics of other sub-Saharan African countries. In addition, to provide an insight into using the FARA country studies as a starting point.

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7 Appendix 1: Sector Classifications

No.	Code	No.	Code	Description	No.	Code	No.	Code	Description
1	Agriculture	1	pdr	Paddy rice	3	Service	47	trd	Trade
		2	wht	Wheat			48	otp	Transport nec
		3	gro	Cereal grains nec			49	wtp	Sea transport
		4	v_f	Vegetables, fruit, nuts			50	atp	Air transport
		5	osd	Oil seeds			51	cmn	Communication
		6	c_b	Sugar cane, sugar beet			52	ofi	Financial services nec
		7	pfb	Plant-based fibers			53	isr	Insurance
		8	ocr	Crops nec			54	obs	Business services nec
		9	ctl	Cattle,sheep,goats,horses			55	ros	Recreation and other services
		10	oap	Animal products nec			56	osg	PubAdmin/Defence/Health/Educat
		11	rmk	Raw milk			57	dwe	Dwellings
		12	wol	Wool, silk-worm cocoons					
		13	frs	Forestry					
		14	fsh	Fishing					
2	Industry	15	coa	Coal			31	ppp	Paper products, publishing
		16	oil	Oil			32	p_c	Petroleum, coal products
		17	gas	Gas			33	crp	Chemical,rubber,plastic prods
		18	omn	Minerals nec			34	nmm	Mineral products nec
		19	cmt	Meat: cattle,sheep,goats,horse			35	i_s	Ferrous metals
		20	omt	Meat products nec			36	nfm	Metals nec
		21	vol	Vegetable oils and fats			37	fmp	Metal products
		22	mil	Dairy products			38	mvh	Motor vehicles and parts
		23	pcr	Processed rice			39	otn	Transport equipment nec
		24	sgr	Sugar			40	ele	Electronic equipment
		25	ofd	Food products nec			41	ome	Machinery and equipment nec
		26	b_t	Beverages and tobacco products			42	omf	Manufactures nec
		27	tex	Textiles			43	ely	Electricity
		28	wap	Wearing apparel			44	gdt	Gas manufacture, distribution
		29	lea	Leather products			45	wtr	Water
		30	lum	Wood products			46	cns	Construction

8 Appendix 2: Growth Accounting and TFP

8.1 Total factor productivity

This section provides more details on the growth accounting approach to measuring TFP growth. To keep the discussion as close to the actual implementation of the country studies, the discussion (and notation) below departs slightly from that found in the monograph and the country reports. Let Y_t represent real GDP in period t, and let K_t and L_t represent capital and labor at time t.[7] Next, let $B_t = a_o + a_t$ capture the baseline level of technology, a_o, and non-systematic shocks to production, a_t. Finally, let $A_t = (1 + x_t)A_{t-1}$ represent augments to labor productivity, where x_t represents the (exogenous) Harrod neutral rate of technical change.

A country's aggregate production technology is given by the Cobb-Douglas function

(3) $$Y_t = B_t K^{\alpha}(A_t L_t)^{1-\alpha}$$

where the factor share parameter, α, satisfies $\alpha \in (0,1)$. To get an expression of total factor productivity growth, take the log of equation (3)

$$\ln Y_t = \ln B_t + \alpha \ln K_t + (1-\alpha)\ln L_t + (1-\alpha)\ln A_t$$

and first difference the above equation to get

$$\ln Y_t - \ln Y_{t-1} = \ln B_t - \ln B_{t-1} + \alpha(\ln K_t - \ln K_{t-1}) + (1-\alpha)(\ln L_t - \ln L_{t-1}) + (1-\alpha)(\ln A_t - \ln A_{t-1})$$

Next, use the approximation result $\ln X_t - \ln X_{t-1} \cong \frac{X_t - X_{t-1}}{X_{t-1}}$ and make the appropriate substitutions to get the Solow measure of total factor productivity growth

(4) $$TFP_t = \frac{a_t - a_{t-1}}{a_{t-1}} + (1-\alpha)x_t = \frac{Y_t - Y_{t-1}}{Y_{t-1}} - \alpha\frac{K_t - K_{t-1}}{K_t} + (1-\alpha)\frac{L_t - L_{t-1}}{L_t}$$

Note that TFP_t is the Solow residual, and the country studies call the expression, $(1-\alpha)x_t$, "Solow TFP."

The data requirements for calculating the Solow residual – or Solow TFP – are are inferred from Equation (4). One needs time series data on real GDP, capital stock and labor force levels, and data on

[7] The country studies and FARA monograph view labor as "human capital," denoted H_t, where human capital is defined as $H_t = L_t P_t e^{0.1S_t}$. The variable P_t is the participation rate (share of working age population actually working) and $e^{0.1S_t}$ is the Mincer equation defined over average years of schooling, S_t.

The data requirements for calculating the Solow residual – or Solow TFP – are are inferred from Equation (4). One needs time series data on real GDP, capital stock and labor force levels, and data on capital's cost share. Although relatively simple to implement, TFP estimates based on the production function specified in equation (3) are sensitive to various factors. The Uganda report summarizes these nicely, and are repeated below:

1. Imperfect competition. If the labor market for example is not perfect, then the marginal productivity of labor cannot be reflected by the wage rate. When imperfections exist in the labor market, the wage rate is not a true reflection of the quality/level of skills of labor (Groth, Gutierrez-Domenech and Srinivasan, 2004).

2. How inputs are measured. For example Groth, Gutierrez-Domenech and Srinivasan, (2004) show how measuring capital as the stock of capital instead of the flow of services that capital stock generates is an inaccurate measure of productivity. They suggest the capital services measure which uses different assets weighted together by their rental price weights instead of using the capital stock measure where different stocks of assets are weighted by their market price weights. The rental price is the price that a user would have to pay to rent the asset for a period of time and, in a competitive market, it will reflect the value of the services which can be derived from the asset.

3. Quality of inputs with particular attention to human capital. For example Groth, Gutierrez-Domenech and Srinivasan (2004) show how measuring labor input simply as total hours of work disregards the fact that hours of work are not homogeneous and show how the education composition of the workforce has a bearing on the quality of labor. The Mincer equation attempts to solve this but the growth accounting formula does not consider effort.

4. Factor shares in real GDP and, while shares are constant in the Cobb-Douglas specification, they are not so in other linearly homogenous functional forms such as the Constant Elasticity of Substitution (CES) production function.

8.2 Labor productivity

We are often concerned with labor productivity, e.g., the question posed by Herrendorf and Schoellman (2014): why is measured labor productivity so low in agriculture?[8] Labor productivity is defined as DGP/L where L is often measured as hours worked. How does the growth accounting discussed above compare to d (GDP/L)/dt? Return to the GDP function (7). We established that

[8]They identify three sources for underestimation of agricultural GDP: (1) the payments to farm contractors are classified in agricultural services, (2) the rental payments to land owners tend to be classified in real estates and (3) under-reporting of properties' income.

$$\frac{\dot{Y}}{Y} = S_K \frac{\dot{K}}{K} + (1 - S_K)\left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right)$$

The growth *rate* in real GDP per worker (or hours worked) is

$$\frac{d}{dt} Ln\left[\frac{Y}{L}\right] = \frac{\dot{Y}}{Y} - \frac{\dot{L}}{L}$$

Thus, subtract $\dot{L}/L$ from both sides to obtain

$$\frac{\dot{Y}}{Y} - \frac{\dot{L}}{L} = S_K \frac{\dot{K}}{K} + S_K\left(\frac{\dot{A}}{A} + \frac{\dot{L}}{L}\right) - \frac{\dot{L}}{L} \Rightarrow$$

$$= (1 - S_K)\frac{\dot{A}}{A} + S_K\left(\frac{\dot{K}}{K} - \frac{\dot{L}}{L}\right)$$

The last term in brackets is the rate of growth in the capital-labor ratio *K/L*. The neoclassical growth model predicts that a country converging "from below" to its long-run equilibrium should experience $\dot{K}/K \geq \dot{L}/L$. We refer to $\dot{K}/K \geq \dot{L}/L$ as *capital deepening*.

9 Capital Stock Measurement - Economywide

Measuring the stock of capital for a sector a data and labor intensive undertaking. In fact, the use of the term here is misleading, in that we are not about to discuss how to measure (the value of) capital stocks – we will discuss how to construct a capital stock series from secondary data sources. Below we discuss three approaches to constructing a capital stock series, each of which use what Hall and Jones (1994) call the "perpetual inventory method." The perpetual inventory method is based on the following expression

(5)
$$K_t = K_{t-1} - \delta K_{t-1} + I_{t-1}, \quad t = 1, \dots, \tau^*$$

Here, K_t is the level of the capital stock in period t, K_{t-1} is the level of the capital stock in the prior period, I_{t-1} is gross investment in the prior period and δ is the rate at which capital depreciates each period (assumed constant over time). The parameter τ^* represents the final time period over which the capital stock series will be constructed.

In practice, analysts proxy gross investment by gross fixed capital formation (GFK), and typically download the series from the World Bank's World Development Indicators. Given GFK data, it turns out that equation (5) is a recursive expression whose solution only requires an initial capital stock level, denoted K_0: in other words, it is a system of τ^* equations in $\tau^* + 1$ unknows, i.e., an equation for each capital stock level in periods 1,2, … τ^*, but not for the initial capital stock level, K_0. The distinction between the three methods used to construct a capital stock series lay in how one chooses the initial capital stock. We now review these three methods.

9.1 **Method I**

The first method we discuss is the simplest to implement and is easily executed in an Excel worksheet. To begin, subtract K_{t-1} from both sides of equation (5) to get

(6) $$K_t - K_{t-1} = -\delta K_{t-1} + GFK_{t-1}$$

Observe that $\frac{K_t - K_{t-1}}{K_{t-1}} = g_K$ where g_K is simply the rate of growth in the capital stock between periods t and t-1. Using this observation, rewrite equation (6) as

$$g_K K_{t-1} = -\delta K_{t-1} + GFK_{t-1}$$

or

$$K_t = \frac{GFK_t}{g_K + \delta}$$

Hence, one estimate of the initial capital stock is derived by dividing gross fixed capital formation in the initial period by the rate of depreciation and a rate of growth in the capital stock. If the analyst's time series run from 1970 through 2013, he or she might set g_K equal to the average rate of GDP growth over that period. The reasoning here is that in the long run, with balanced growth, the rate of growth in GDP is equal to the rate of growth in the capital stock. One might also try the average rate of growth in gross fixed capital formation. See the Monograph or any of the country reports for more on the choice of g_K.

9.2 **Method II: The "Kehoe" approach**

Method I has a potentially undesirable feature in that the growth rate used to calculate the initial capital stock is assumed to be a long run growth rate, but whose value is calculated using transition path growth data. The next two subsections circumvent this assumption using a couple clever identities observed by Professor Timothy Kehoe. See "How to construct capital stock and parameters" at http://www.econ.umn.edu/ ~ tkehoe/computation.html. One approach assumes an exogenous depreciation rate and calculates an initial capital stock and accompanying capital stock series. The approach estimates both the rate of depreciation and the initial capital stock endogenously. Of course, we take responsibility for any errors and misinterpretation of these notes.

9.2.1 Exogenous depreciation

Kehoe suggests closing the system using the following condition

$$\frac{K_{\tau_0+1}}{K_{\tau_0}} = \left(\frac{K_{\tau_0+\tau^*}}{K_{\tau_0}}\right)^{1/\tau^*}$$

Taking the log of both sides of this expression yields

(7) $$Ln\left[K_{\tau_0+1}\right] - Ln\left[K_{\tau_0}\right] = \frac{1}{\tau^*}\left(Ln\left[K_{\tau_0+\tau^*}\right] - Ln\left[K_{\tau_0}\right]\right)$$

Equation (7) requires that the initial capital stock be chosen so its rate of growth in the initial period be equal to the average rate of capital stock growth across all periods. Given a gross fixed capital series and a depreciation rate, the recursive system is now given by

$$\begin{gathered} K_1 = (1-\delta)K_0 + I_0 \\ K_2 = (1-\delta)K_1 + I_1 \\ \vdots \\ K_{\tau^*} = (1-\delta)K_{\tau^*-1} + I_{\tau^*-1} \\ Ln\left[K_{\tau_0+1}\right] - Ln\left[K_{\tau_0}\right] = \frac{1}{\tau^*}\left(Ln\left[K_{\tau_0+\tau^*}\right] - Ln\left[K_{\tau_0}\right]\right) \end{gathered}$$

The solution of which yields an initial capital stock level, K_0, and capital stock series consistent with the growth rate conditions required in (7). This system is readily solved in Mathematica or other similar numerical solution software. See the Monograph or any of the country reports (except Tunisia and Morocco) for a discussion on how to solve this system in Excel.

One point we should note here, is the closure rule (7) is more or less, arbitrary. Another closure rule – not discussed in Roe, Smith and Choi (2015) or country reports, but suggested by Kehoe – is

(7') $$\frac{K_0}{Y_0} = \sum\nolimits_{t=0}^{\tau^*} \frac{K_t}{Y_t}$$

9.2.2 Endogenous depreciation

Given a gross fixed capital series and exogenous level of depreciation, we needed one additional equation to close the recursive system of equations (5). When searching for an endogenous depreciation rate, we need two additional equations to close the system: one related to the capital stock levels and the other tied to depreciation rates. With As was the case with exogenous depreciation, the rule in this case is to choose an initial K_0 say $\overline{K}_{\tau_0}$, such that (16) holds. Again, as in the previous section, the growth in capital stock in τ_0 to $\tau_0 + 1$ is based on an assumption that the growth path of the capital stock is growing at an average rate, n. However, unlike previous section, the availability of depreciation data (D_t) modifies the law of motion as follows

$$K_{t-1} = K_t - D_t + I_t \qquad (24)$$

Using the law of motion and going back, say, τ^*=10 years and then setting τ= 0, we find that the ratio of capital between periods 0 and 1 is the same as (16).

$$\frac{K_1}{K_0} = \left(\frac{K_{10}}{K_0}\right)^{1/10} \qquad (25)$$

Using the law of motion (24), we write K_1 as:

$$K_1 = K_0 - D_0 + I_0 \qquad (26)$$

where I_0 and D_0 are investment and depreciation in period 0, respectively. Dividing both sides of (26) by K_0 we get:

$$\frac{K_{10}}{K_0} = 1 - \frac{D_0}{K_0} + \frac{I_0}{K_0} \qquad (27)$$

Based on the preceding, we then proceed as follows: Using the law of motion (24), write K_{10} as:

$$\begin{aligned} &K_{10}= \\ &K_9 - D_9 + I_9 = (K_8 - D_8 + I_8) - D_9 + I_9 = \cdots = \\ &K_0 - \{D_0 + D_1 + \cdots + D_9\} + \{I_0 + I_1 + \cdots + I_9\} \qquad (28) \end{aligned}$$

The above expression can be written in general terms as:

$$K_{\tau_0+\tau^*} = K_{\tau_0} - \sum_{t=1}^{\tau^*} D_{\tau_0+t-1} + \sum_{t=1}^{\tau^*} I_{\tau_0+t-1} \qquad (29)$$

Returning to expression (28), and dividing it through by K_0 to get:

$$\frac{K_{10}}{K_0} = 1 - \frac{D_0 + D_1 + \cdots + D_9}{K_0} + \frac{I_0 + I_1 + \cdots + I_9}{K_0} \qquad (30)$$

As before, note that the RHS of (30) equals $\frac{K_{10}}{K_0}$ in (25).

Returning to (25) and substituting (27) for K_{1/K_0} and (25), and (30) for K_{10}/K_0 we obtain.

$$1 - \frac{D_0}{K_0} + \frac{I_0}{K_0} = (1 - \frac{D_0 + D_1 + \cdots + D_9}{K_0} + \frac{I_0 + I_1 + \cdots + I_9}{K_0})^{1/10}$$

Or, the "Kehoe" equation becomes, for $\tau^* = 10$,

$$1 - \frac{D_0}{K_0} + \frac{I_0}{K_0} = \left[1 - \frac{\sum_{t=1}^{10} D_{t-1}}{K_0} + \frac{\sum_{t=1}^{10} I_{t-1}}{K_0}\right]^{1/10} \tag{31}$$

Notice, that we have reduced the equation to one unknown K_0, given data I_t and D_t. It remains, now, to write code numerically to find the root K_0 satisfying equation (31). Then, we can use the law of motion (24) or equivalently, its counterpart (29), to calculate the remaining K_t.

More generally, to estimate $\left\{K_{\tau_{0,}} K_{\tau_0+1}, \dots, K_{\tau_0+\tau^*}\right\}$ over a reference period $\{\tau_0, \tau_0+1, \dots, \tau_0+\tau^*\}$ given data $\{(I_{\tau_0}, D_{\tau_0}), \dots, (I_{\tau_0+\tau^*-1}, D_{\tau_0+\tau^*-1})\}$, we write code to find the root K_{τ_0} using

$$1 - \frac{D_{\tau_0}}{K_{\tau_0}} + \frac{I_{\tau_0}}{K_{\tau_0}} = \left[1 - \frac{\sum_{t=1}^{\tau^*} D_{\tau_0+t-1}}{K_{\tau_0}} + \frac{\sum_{t=1}^{\tau^*} I_{\tau_0+t-1}}{K_{\tau_0}}\right] \tag{32}$$

Given an estimate of initial capital stock $K(\tau_0)$ from (32), either the equation of motion (24) or (29) is used to calculate the remaining sequence $\{K(\tau_0+t)\}$, t = 1, 2, ..τ^*.

Finally, we calculate an estimate of annual depreciation rate,$\delta(t)$, using data on investment and depreciation as well as capital stock estimated above.

Suppose, albeit counterfactually, that the depreciation rate, δ, is roughly constant over the reference period. Then the constant depreciation rate would have satisfied the following equation.

$$K_{\tau_0}(1-\delta)^{\tau^*} + \sum_{t=1}^{\tau^*}(1-\delta)^{\tau_0+\tau^*-t} I_{\tau_0+t-1} = K_{\tau_0+\tau^*} \tag{33}$$

The LHS of (33) implies the level of capital stock at the end of period $\tau_0 + \tau^*$based on the law of motion (17) for the case of exogenous depreciation rate, given the initial K_{τ_0}, (estimated above) and constant) and a constant rate of depreciation, δ. The RHS is the capital stock at the end of period $\tau_0 + \tau^*$ estimated above. Given the data on I_τ and estimated $\{K(\tau_0), K(\tau_0+\tau^*)\}$, this is an equation with one unknown, δ. It remains, now, to write code to find, numerically, the root δ satisfying (33).

10 Sector Level Capital Stocks and Sector TFP

If willing to assume capital markets are well functioning (complete) and clear, apportioning a capital stock across competing sectors is relatively straightforward. To illustrate how this is done, consider the following aggregate production function

$$Y_t = 25K_t^{0.5}L_t^{0.5}$$

The marginal value product of capital, denoted *MVP*$_K$ is

$$MVP_K = 12.5\left(\frac{L_t}{K_t}\right)^{0.5}$$

and in equilibrium, $MVP_K = r^k$, where r^k is the rate of return to capital adjusted for depreciation. If $K_{2010} = 100{,}000{,}000$ and $L_{2010} = 2{,}500$, then $Y_{2010} = 12{,}500{,}000$ and $r^k = 0.065$.

Next, as in each country study, assume your social accounting matrix (or input-output table) is aggregated into three sectors, agriculture, manufacturing and services, and denote agricultural, manufacturing and service sector value added by $Y_{a,t}$, $Y_{m,t}$ and $Y_{s,t}$, respectively. Further assume agriculture's share of value added is $\sigma_a = 0.20$, while manufacturing and service's share of value added are $\sigma_m = 0.30$ and $\sigma_s = 0.50$, respectively. Then

$$Y_{j,t} = \sigma_j Y_t, \qquad j = a, m, s$$

Finally, assume the sector capital cost shares are $\alpha_a = 0.35$, $\alpha_m = 0.65$ and $\alpha_s = 0.4$ for the agricultural, manufacturing and services sectors respectively.

Capital market clearing implies the marginal value product of capital is equal across the three sectors. The reader can show that (suppressing the time subscripts) with Cobb-Douglas sector production functions the following condition holds in equilibrium

$$\frac{\alpha_a Y_a}{K_a} = \frac{\alpha_m Y_m}{K_m} = \frac{\alpha_s Y_s}{K_s} = r^k$$

or

$$K_j = \frac{\alpha_j Y_t}{r^k}, \qquad j = a, m, s$$

Armed with the sector capital stock series, labor levels, value added levels and cultivated area, one calculates sector-*j* TFP growth rates as

$$TFP_{j,t} = \frac{a_{j,t} - a_{j,t-1}}{a_{j,t-1}} + (1-\alpha)x_{j,t} = \frac{Y_{j,t} - Y_{j,t-1}}{Y_{j,t}} - \alpha\frac{K_{j,t} - K_{j,t-1}}{K_{j,t}} + (1-\alpha)\frac{L_{j,t} - L_{j,t-1}}{L_{j,t}}$$

where the notation should have obvious interpretations.

11 The Hodrick-Prescott filter

The careful reader may have observed our use of the Hodrick-Prescott filter. We do this because the Solow residual tends to track output growth closely, and since $a(t)$ likely varies over time, those variations will be embodied in the TFP estimates. We use the TFP trend provided by the Hodrick-Prescott filter to calculate TFP as a preferred estimate of $(1-\alpha)x$. The term $a(t)$ may include changes in adjudication, infrastructure, weather, trade shocks and other factors. For this reason, some have regressed Solow's residual (or the deviations from the filtered values) on variables (Gopinath and Roe, 1997) to help identify the components causing variation in the $a(t) + (1-\alpha)\,x$ term. We apply the Hodrick-Prescott filter (1997). The equation is given by

$$\underset{\{t_\tau\}_{\tau^*}^{1}}{Min} \left\{\sum_{\tau=1}^{\tau^*}(TFP_\tau - t_\tau) + \lambda \sum_{\tau=1}^{\tau^*-1}[(t_\tau - t_{\tau-1}) - (t_\tau - t_{\tau-2})]^2\right\} \qquad (9)$$

where t_τ is the trend component of the residual TFP series. This filter has the property that mean of the filtered series t_τ is the same as the mean of the unfiltered series TFP_t. The term λ is a Lagrangian like multiplier that must be chosen, and treated as an exogenous constant. For annual data, some authors recommend a value of 6.25 (which is used in our analysis below), others suggest a range of values $6 < \lambda < 14$.

As mentioned in our discussion of the World Bank Note above, **a**(*t*) in (1) is some noise-residual variable that can be problematic to explain. In our empirical analysis here, we "filter" Solow's residual in an attempt to identify Solow TFP. We apply the Hodrick-Prescott filter (1997). The equation is given by

$$\underset{\{t_\tau\}_{\tau^*}^{1}}{Min} \left\{\sum_{\tau=1}^{\tau^*}(TFP_\tau - t_\tau) + \lambda \sum_{\tau=1}^{\tau^*-1}[(t_\tau - t_{\tau-1}) - (t_\tau - t_{\tau-2})]^2\right\} \qquad (9)$$

where t_τ is the trend component of the residual TFP series. This filter has the property that mean of the filtered series t_τ is the same as the mean of the unfiltered series TFP_t. The term λ is a Lagrangian like multiplier that must be chosen, and treated as an exogenous constant. For annual data, some authors recommend a value of 6.25 (which is used in our analysis below), others suggest a range of values $6 < \lambda < 14$.

About FARA

The Forum for Agricultural Research in Africa (FARA) is the apex continental organization responsible for coordinating and advocating for agricultural research-for-development. (AR4D). It serves as the entry point for agricultural research initiatives designed to have a continental reach or a sub-continental reach spanning more than one sub-region.

FARA serves as the technical arm of the African Union Commission (AUC) on matters concerning agricultural science, technology and innovation. FARA has provided a continental forum for stakeholders in AR4D to shape the vision and agenda for the sub-sector and to mobilise themselves to respond to key continent-wide development frameworks, notably the Comprehensive Africa Agriculture Development Programme (CAADP).

FARA's vision: Reduced poverty in Africa as a result of sustainable broad-based agricultural growth and improved livelihoods, particularly of smallholder and pastoral enterprises.

FARA's mission: Creation of broad-based improvements in agricultural productivity, competitiveness and markets by continental-level strengthening of capacity for agricultural innovation.

FARA's value proposition: Strengthening Africa's capacity for innovation and transformation by visioning its strategic direction, integrating its capacities for change and creating an enabling policy environment for implementation.

FARA's strategic direction is derived from and aligned to the Science Agenda for Agriculture in Africa (S3A), which is, in turn, designed to support the realisation of the CAADP vision. FARA's programme is organised around three strategic priorities, namely:

- Visioning Africa's agricultural transformation with foresight, strategic analysis and partnerships to enable Africa to determine the future of its agriculture, with proactive approaches to exploit opportunities in agribusiness, trade and markets, taking the best advantage of emerging sciences, technologies and risk mitigation and using the combined strengths of public and private stakeholders.
- Integrating capacities for change by making the different actors aware of each other's capacities and contributions, connecting institutions and matching capacity supply to demand to create consolidated, high-capacity and effective African agricultural innovation systems that can use relative institutional collaborative advantages to mutual benefit while also strengthening their own human and institutional capacities.
- Enabling environment for implementation, initially through evidence-based advocacy, communication and widespread stakeholder awareness and engagement and to generate enabling policies, and then ensure that they get the stakeholder support required for the sustainable implementation of programmes for African agricultural innovation

Key to this is the delivery of three important results, which respond to the strategic priorities expressed by FARA's clients. These are:

Key Result 1: Stakeholders empowered to determine how the sector should be transformed and undertake collective actions in a gender-sensitive manner

Key Result 2: Strengthened and integrated continental capacity that responds to stakeholder demands within the agricultural innovation system in a gender-sensitive manner

Key Result 3: Enabling environment for increased AR4D investment and implementation of agricultural innovation systems in a gender-sensitive manner

FARA's development partners are the African Development Bank (AfDB), the Canadian International Development Agency (CIDA)/ Department of Foreign Affairs, Trade and Development (DFATD), the Danish International Development Agency (DANIDA), the Department for International Development (DFID), the European Commission (EC), The Consultative Group in International Agricultural Research (CGIAR), the Governments of the Netherlands and Italy, the Norwegian Agency for Development Cooperation (NORAD), Australian Agency for International Development (AusAiD) and The World Bank.

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